PROBLEM SET 3: COMPUTING LIMITS

Note: Most of the problems were taken from the textbook [1].

Problem 1. *Find the equation of a tangent line to the graph of the function* $f(x) = x^2 + 4x + 3$ *passing by the point* $P = (0, -2)$.

Problem 2. *Use the graph of the function* $f(x) = \frac{x^2+x}{\sqrt{x^3+x^2}}$ *to state the value of each limit if it exists. If it does not exists explain way.*

a) $\lim_{x\to 0^-} f(x)$;

b) $\lim_{x\to 0^+} f(x)$;

c) $\lim_{x\to 0} f(x)$.

Do the same if $f(x) = (1 + e^{1/x})^{-1}$.

Problem 3. *Sketch a graph of an example of a function* f *that satisfies all of the following conditions:*

a) $\lim_{x\to 0^-} f(x) = 2$;

b) $\lim_{x\to 0^+} f(x) = 0$;

c) $\lim_{x\to 4^-} f(x) = 3$;

d) $\lim_{x\to 4^+} f(x) = \infty$;

e) $f(0) = 2$;

f) $f(4)$ *is not defined.*

Problem 4. *Determine the infinite limits:*

a) $\lim_{x\to 0^+} \ln(\sin x)$;

b) $\lim_{x\to \pi^-} \cot x$;

c) $\lim_{x\to 2^+} \frac{x^2-2x-8}{x^2-5x+6}$.

d) $\lim_{x\to 0^+} \left(\frac{1}{x} - \ln x\right)$.

Problem 5. *Evaluate each of the following limits, it it exists:*

a) $\lim_{x\to 1} \frac{x^4-1}{x^3-1}$;

b) $\lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t}$;

c) $\lim_{x\to -4} \frac{\sqrt{x^2+9}-5}{x+4}$.

d) $\lim_{h\to 0} \frac{(x+h)^3-x^3}{h}$.

Problem 6. *If* $2x \leq g(x) \leq x^4 - x^2 + 2$ *for all* x*, evaluate* $\lim_{x\to 1} g(x)$.

Problem 7. *Use the Squeeze Theorem to show that*

$$\lim_{x\to 0} \sqrt{x^3 + x^2} \sin \frac{\pi}{x} = 0.$$

Problem 8. *Prove that* $\lim_{x\to 0} x^4 \cos \frac{2}{x} = 0$.

Problem 9. *Find each of the following limits if it exists. If it does not exist, explain why.*

a) $\lim_{x\to -6} \frac{2x+12}{|x+6|}$;

b) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{|x|} \right)$.

References

[1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.