Course 18.211: Combinatorial Analysis MIT, Fall 2021

Class Meetings: Mondays/Wednesdays/Fridays 11-12pm; Room: 4-163

Instructor: Felix Gotti (Email Address: fgotti@mit.edu; Office: 2-232C)

Office Hours: TBD

Prereq. Calculus II (GIR) and (18.06, 18.700, or 18.701)

Description. This course is intended to be an introduction to combinatorial theory. The first part of the course is devoted to enumeration, construction of bijections, and generating functions. The second part of this course is mostly an introduction to graph theory and, if times permits, some aspects of partially ordered sets (posets) and lattices will be covered.

Textbooks. The recommended textbook for this class is

- *A Walk Through Combinatorics* by Miklos Bona (any edition of the book will be suitable for this course).

Although it will not be required, for deeper knowledge or further related topics, students can consult/read the following books:

- *Enumerative Combinatorics* (Vol. I) by Richard Stanley,
- *Algebraic Combinatorics: Walks, Trees, Tableaux, and More* by Richard Stanley,
- *Modern Graph Theory* by Béla Bollobás.

Grading Policy. There will be 4 in-class 1-hour midterms, and 3 of them will contribute 60% ($3 \times 20\%$) of the final grade (the one with the lowest grade will be dropped). There will also be 6 problem sets, and 5 of them will contribute 40% ($5 \times 8\%$) towards the final grade (the one with the lowest grade will be dropped).

Midterms. Midterms will take place in class. They will be closed-book exams (and notes, calculators, and electronic devices are not allowed). Each midterm will cover roughly a quarter of the material of this course (all the lectures covered after the previous midterm). Practice midterms, along with their solutions, will be posted in the course website at least a week before the corresponding midterm. There will not be make-up exams as students are allowed to drop one of the four midterms without penalty.

Problem Sets. Problem sets will be assigned systematically throughout the course. Students are allowed to collaborate in a constructive and reasonably manner. Each student is expected to write down solutions in her/his own words showing complete understanding of each turned-in solution. Copying solutions from another student/source is considered cheating/plagiarism. In case of collaboration, students should list the names of their collaborators as part of their solutions. Note that typed or understandable handwritten solutions are expected, and every step of each solution should be clearly justified. Solutions of problem sets should be turned in at the beginning of the lecture of the corresponding due date (late solutions to problem sets will not be accepted in the absence of a proper documented excuse).

1. Problem Set 1 (Due on Wednesday, 09/22)
2. Problem Set 2 (Due on Wednesday, 10/13)
3. Problem Set 3 (Due on Wednesday, 10/27)
4. Problem Set 4 (Due on Monday, 11/08)
5. Problem Set 5 (Due on Monday, 11/22)
6. Problem Set 6 (Due on Wednesday, 12/08)

Tentative Schedule of Topics

1. (W 09/08). Pigeonhole Principle
2. (F 09/10). Mathematical Induction
3. (M 09/13). Elementary Counting
4. (W 09/15). Binomial and Multinomial Theorems
5. (F 09/17). Inversions of Permutations, q-Factorials, and q-Binomials
6. (M 09/20). Compositions
7. (W 09/22). Set Partitions, Stirling Numbers of Second Kind **(Due Date for PS 1)**
8. (F 09/24). Integer Partitions I
9. (M 09/27). Integer Partitions II
10. **(W 09/29). Midterm I (Lectures 1-9)**
11. (F 10/01). Permutations I: Cycle Type
12. (M 10/04). Permutations II: Stirling Numbers of the First Kind
13. (W 10/06). The Sieve Formula
14. (F 10/08). The Problem of Derangements

    (M 10/11). Holiday: Indigenous Peoples Day
15. (W 10/13). Generating Functions I: Fibonacci Numbers **(Due Date for PS 2)**
16. (F 10/15). Generating Functions II: Products and Catalan Numbers

17. (M 10/18). Generating Functions III: Compositions
18. (W 10/20). Exponential Generating Functions I: Recurrences and Products
19. (F 10/22). Exponential Generating Functions II: Exponential Formula
20. **(M 10/25). Midterm II (Lectures 11-19)**

21. (W 10/27). Intro to Graph Theory **(Due Date for PS 3)**
22. (F 10/29). Eulearian Trails and Hamiltonian Cycles
23. (M 11/01). Connectedness and Trees
24. (W 11/03). More on Trees: The Cayley's Formula
25. (F 11/05). Spanning Trees of Graphs
26. (M 11/08). Kruskal's Algorithm and Matroids **(Due Date for PS 4)**
27. (W 11/10). Graph and Matrices: The Matrix-Tree Theorem
28. **(F 11/12). Midterm III (Lectures 21-27)**

29. (M 11/15). Matching in Graphs I: Bipartite Graphs
30. (W 11/17). Matching in Graphs II: Hall's Marriage Theorem
31. (F 11/29). Graph Coloring I: k-Coloring and Brooks' Theorem
32. (M 11/22). Graph Coloring II: Tutte's Theorem **(Due Date for PS 5)**
33. (W 11/24). Planarity I: Euler's Formula and Beyond

    (F 11/26). Institute Holiday
34. (M 11/29). Planarity II: Polytopes
35. (W 12/01). Ramsey Theory
36. **(F 12/03). Midterm IV (Lectures 29-35)**

37. (M 12/06). Intro to Posets and Lattices
38. (W 12/08). The Möbius Inversion Formula **(Due Date for PS 6)**