PROBLEM SET 2: NEW FUNCTIONS FROM OLD FUNCTIONS

Note: Most of the problems were taken from the textbook [1].

Problem 1. *Graph the following functions by hand, not by plotting points, but by starting with the graph of one standard function.*

a) $g(x) = x^2 - 4x + 5$;

b) $f(u) = 3 - 2\cos u$;

c) $h(x) = |\sqrt{x} - 1|$.

Problem 2. *If* $f(x) = |x - 4|$, $g(x) = 2^x$, *and* $h(x) = \sqrt{x}$, *find* $f \circ g \circ h$.

Problem 3. *Express the functions in the form* $f \circ g \circ h$:

a) $R(x) = \sqrt{\sqrt{x} - 1}$;

b) $H(x) = \sqrt[8]{2 + |x|}$;

c) $S(t) = \sin^2(\cos t)$.

Problem 4. *Find the domain of the functions:*

a) $f(x) = \frac{1-e^{x^2}}{1-e^{1-x^2}}$;

b) $h(t) = \frac{1+x}{e^{\cos x}}$;

c) $f(x) = \sqrt{10^x - 100}$.

Problem 5. *Find a formula for the inverse of the function* $\frac{1-e^{-x}}{1+e^{-x}}$.

Problem 6. *Let* $f(x) = \sqrt[5]{3 - x^5}$. *Find the inverse of* $f(x)$. *What can you say about the graph of* $f(x)$ *without having its sketch? Explain.*

Problem 7. *Show that* $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$.

Problem 8. *Simplify the following expressions:*

a) $\tan(\sin^{-1} x)$;

b) $\sin(\tan^{-1} x)$.

References

[1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.