

PROBLEM SET 3: COMPUTING LIMITS

Note: Most of the problems were taken from the textbook [1].

Problem 1. Find the equation of a tangent line to the graph of the function $f(x) = x^2 + 4x + 3$ passing by the point $P = (0, -2)$.

Problem 2. Use the graph of the function $f(x) = \frac{x^2+x}{\sqrt{x^3+x^2}}$ to state the value of each limit if it exists. If it does not exist explain why.

a) $\lim_{x \rightarrow 0^-} f(x)$;

b) $\lim_{x \rightarrow 0^+} f(x)$;

c) $\lim_{x \rightarrow 0} f(x)$.

Do the same if $f(x) = (1 + e^{1/x})^{-1}$.

Problem 3. Sketch a graph of an example of a function f that satisfies all of the following conditions:

a) $\lim_{x \rightarrow 0^-} f(x) = 2$;

b) $\lim_{x \rightarrow 0^+} f(x) = 0$;

c) $\lim_{x \rightarrow 4^-} f(x) = 3$;

d) $\lim_{x \rightarrow 4^+} f(x) = \infty$;

e) $f(0) = 2$;

f) $f(4)$ is not defined.

Problem 4. Determine the infinite limits:

a) $\lim_{x \rightarrow 0^+} \ln(\sin x)$;

b) $\lim_{x \rightarrow \pi^-} \cot x$;

c) $\lim_{x \rightarrow 2^+} \frac{x^2-2x-8}{x^2-5x+6}$.

d) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \ln x \right)$.

Problem 5. Evaluate each of the following limits, if it exists:

a) $\lim_{x \rightarrow 1} \frac{x^4 - 1}{x^3 - 1}$;

b) $\lim_{t \rightarrow 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t}$;

c) $\lim_{x \rightarrow -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}$.

d) $\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$.

Problem 6. If $2x \leq g(x) \leq x^4 - x^2 + 2$ for all x , evaluate $\lim_{x \rightarrow 1} g(x)$.

Problem 7. Use the Squeeze Theorem to show that

$$\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin \frac{\pi}{x} = 0.$$

Problem 8. Prove that $\lim_{x \rightarrow 0} x^4 \cos \frac{2}{x} = 0$.

Problem 9. Find each of the following limits if it exists. If it does not exist, explain why.

a) $\lim_{x \rightarrow -6} \frac{2x+12}{|x+6|}$;

b) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{|x|} \right)$.

REFERENCES

- [1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.