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Ph.D. Qualifying Exam Syllabus

Committee:	Richard Bamler, Bernd Sturmfels (Chair), Nike Sun, Lauren Williams.
Date:	Friday, 13 May 2016 at 1:00 PM
Room:	Evans 891

Major Topic 1: Algebraic Combinatorics (Algebra)

- **Counting:** Basics of generating functions; multisets; representations of permutations: word, standard, inversion table; statistics of permutations: type, inversion, descent, major index; partition identities and Young diagrams; the twelvefold ways.
- **Sieve Methods and their Applications:** Inclusion-Exclusion Principle; permutations with restricted positions: derangements problem and ménage problem; Ferrers boards.
- **Posets and Lattices:** Posets basics; constructions of posets; lattices: modular, complemented, (co)atomic, geometric; distributive lattices; incidence algebras; Möbius Inversion Formula; computing Möbius functions of basic lattices: $[n]$, B_n, D_n; Weisner's theorem and Crosscut Theorem; Möbius functions of semimodular lattices.
- **Generating Functions:** Power series ring; rational generating functions; Hadamard product; polynomials as rational generating functions; quasipolynomials. The exponential formula and its applications; enumeration of trees; the Lagrange Inversion Formula.
- **Symmetric Functions:** The algebra Λ of symmetric functions; partitions and their orderings; monomial, elementary, and complete homogeneous symmetric functions; the involution isomorphism; power sum symmetric functions; specializations; the scalar product of Λ; combinatorial definition of Schur functions.

Reference: R. Stanley, *Enumerative Combinatorics* (Volumes 1 and 2): Ch 1 (sec. 1–4 and 7–9), Ch 2 (sec. 1–4), Ch 3 (sec. 1–10), Ch 4 (sec. 1–4), Ch 5 (sec. 1–4), and Ch 7 (sec. 1–10).

Major Topic 2: Cluster Algebras (Algebra)

- **Total Positivity:** Totally positive matrices; triangulation of polygons and the Grassmannian: the 3-dimensional associahedron; wiring diagrams and flag totally positive.
- **Mutations of Quivers and Matrices:** Quiver mutations; quivers associated to triangulations of polygons and wiring diagrams; urban renewal; mutation equivalence; matrix mutations.
- **Cluster Algebras of Geometric Type:** Basic definitions; rank 1 and 2 cluster algebras; Laurent Phenomenon and its applications; Y-patterns; mutations of Y-seeds and tropical semifields.

- **Constructions of Cluster Algebras:** Seed subpatterns and cluster subalgebras; changing of coefficients; G-admissible seeds and folded seeds.
- **Finite Type Classification:** Finite type classification in rank 2; Cartan matrices and Dynkin diagrams; seed patterns of type A_n; seed patterns of type D_n; seeds patterns o type B_n and C_n; seeds pattern of types E_6, E_7, and E_8; seed patterns of types F_4 and G_2; quasi-Cartan companions.
- **Cluster Structures in Commutative Rings:** Cluster algebras and coordinate rings; example of cluster algebras of classical types; starfish lemma; cluster structure in the ring $\mathbb{C}[\mathrm{SL}_k]^U$; cluster structure in the rings $\mathbb{C}[\mathrm{Mat}_{k\times k}]$ and $\mathbb{C}[\mathrm{SL}_k]$.
- **Triangulated Surfaces:** Bordered surfaces with marked points; ideal triangulations; arc complexes; tagged arc complexes; denominators and intersection numbers; strata and tagged triangulations; mutations and flips in the tagged case; cycles in the graph of flips; tropical exchange relations for intersection numbers.

References: S. Fomin and L. Williams, *Introduction to Cluster Algebras* (under preparation): Ch 1 (sec. 1–3), Ch 2 (sec. 1–3 and 5–8), Ch 3 (sec. 1–6), Ch 4 (sec. 1–3), Ch 5 (sec. 1–10), Ch 7 (sec. 1–5). S. Fomin, M. Shapiro, and D. Thurston. *Cluster Algebras and Triangulated Surfaces: Cluster Complexes*: Sec. 1–9.

Minor Topic: Differential Topology (Topology)

- **Manifolds:** Definition of manifolds; smooth maps on a manifold; partial derivatives; the Inverse Function Theorem; quotients; the real projective space as a manifold.
- **The Tangent Space:** The tangent space; submanifolds; the Regular Level Set Theorem; immersions and submersions; the immersion and submersion theorem; the tangent/vector bundle; partition of unity; vector fields; integral curves; local flows; Lie bracket.
- **Lie Groups and Lie Algebras:** Example of Lie groups; Lie subgroups; Lie group homomorphisms; the matrix exponential. Example of Lie algebras; the Lie algebra of a Lie group; left-invariant vector fields and their pushforwards; the differential as a Lie algebra homomorphism.
- **Differential Forms:** Differential 1-forms: the cotangent bundle; differential k-forms: their pullback and wedge product; the exterior derivative; the Lie derivative and the interior multiplication; invariant forms on a Lie group.
- **Integration:** Orientations on a manifold; orientations and differential forms; manifolds with boundary; integration on manifolds; Stokes's theorem; line integral and Green's theorem.
- **De Rham Cohomology** Definition of De Rham cohomology; examples of de Rham cohomology; diffeomorphism invariance; the ring structure on de Rham cohomology

Reference: L. Tu, *An Introduction to Manifolds*, 2nd edition: Ch. 1–7 (sec. 1–24).